

# Thermal Leptogenesis in the Minimal Gauged $U(1)_{L_\mu - L_\tau}$ Model

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SCHOOL OF SCIENCE  
THE UNIVERSITY OF TOKYO

Based on JHEP 09 (2023) 079 [hep-ph 2305.18100]

Alessandro Granelli, Koichi Hamaguchi, Natsumi Nagata, Maura E. Ramirez-Quezada, and JW

# Leptogenesis

M. Fukugita, T. Yanagida Phys. Lett. B 174 45-47 (1986)

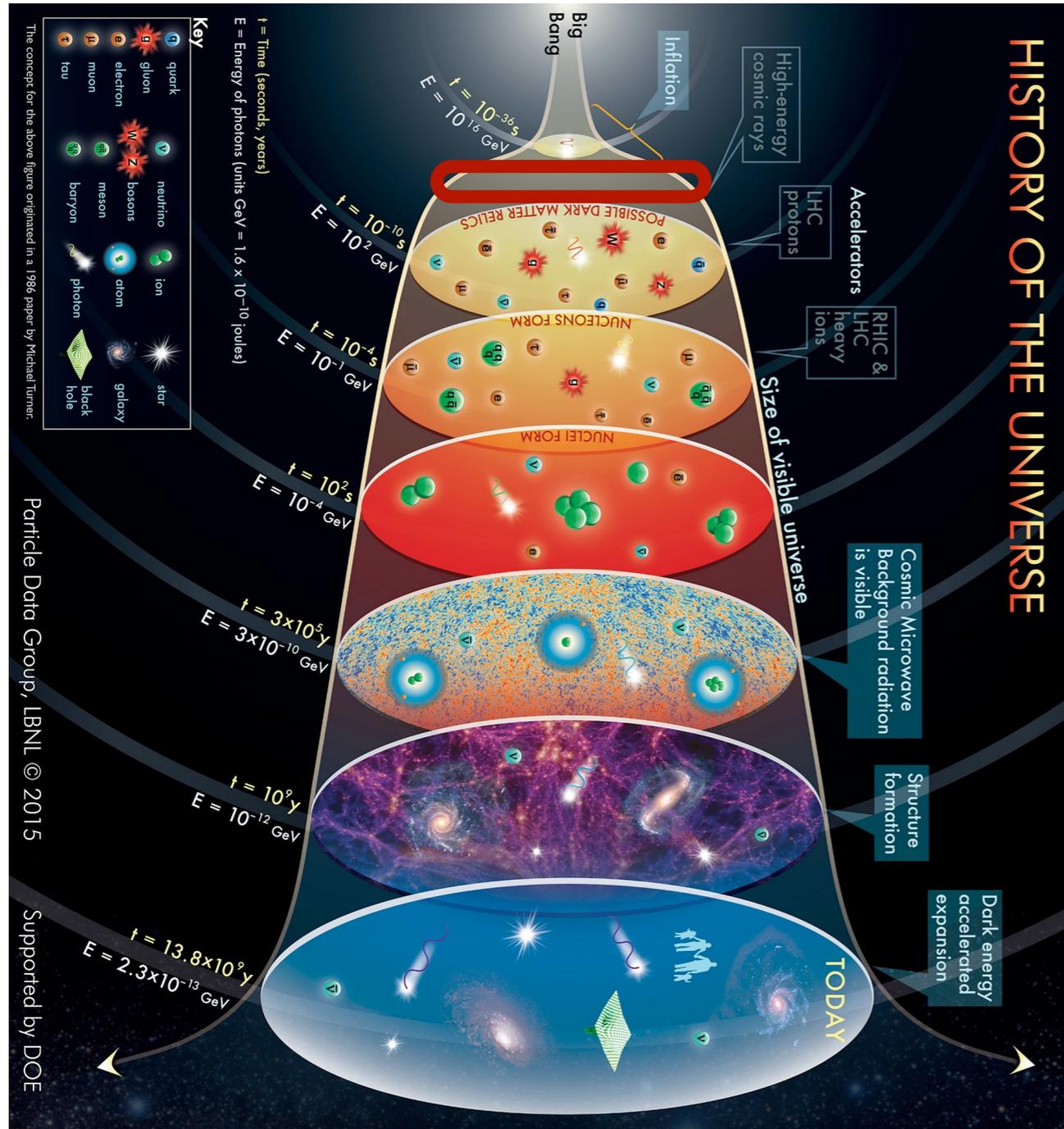
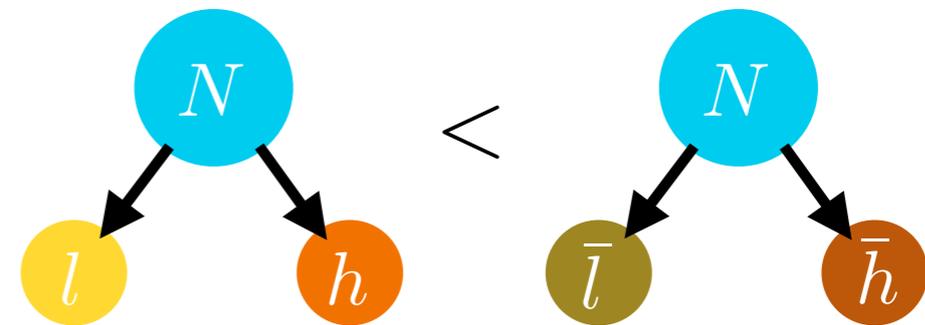


Fig from PDG

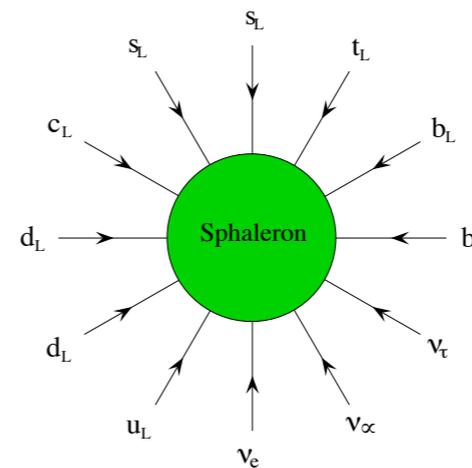
## RH $\nu$ decay



$> 10^{10}$  GeV

## Sphaleron process

V.A. Kuzmin et al., Phys. Rev. B 155 36-42 (1985)

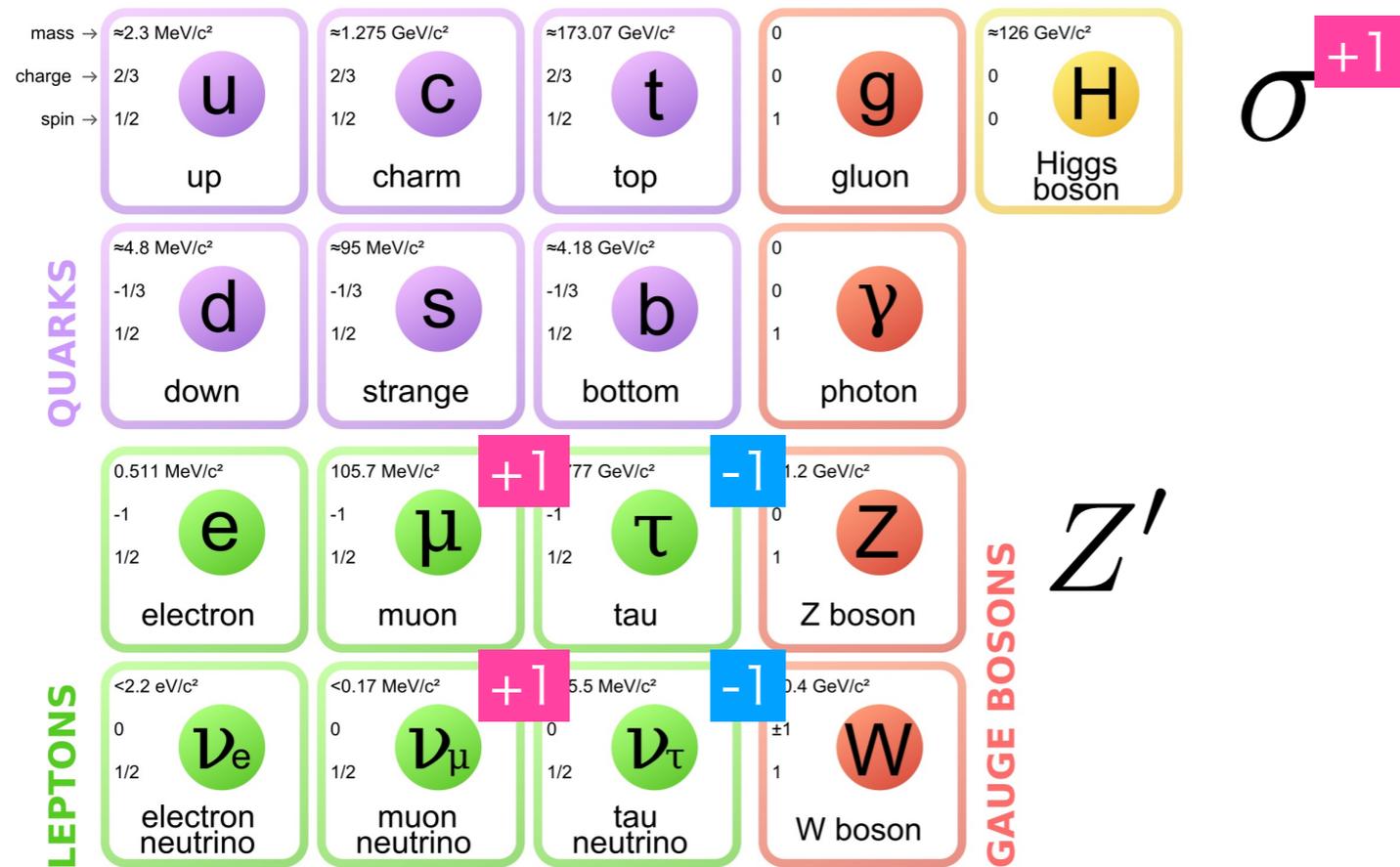


$> 10^3$  GeV

Fig from W. Buchmüller, Nucl. Phys. B Proc.Suppl. 235-236 329-335 (2013)

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

## Minimal Setup



$$N_e, N_\mu, N_\tau$$

QUANTUM DIARIES  
<https://www.quantumdiaries.org/2014/03/14/the-standard-model-a-beautiful-but-flawed-theory/>

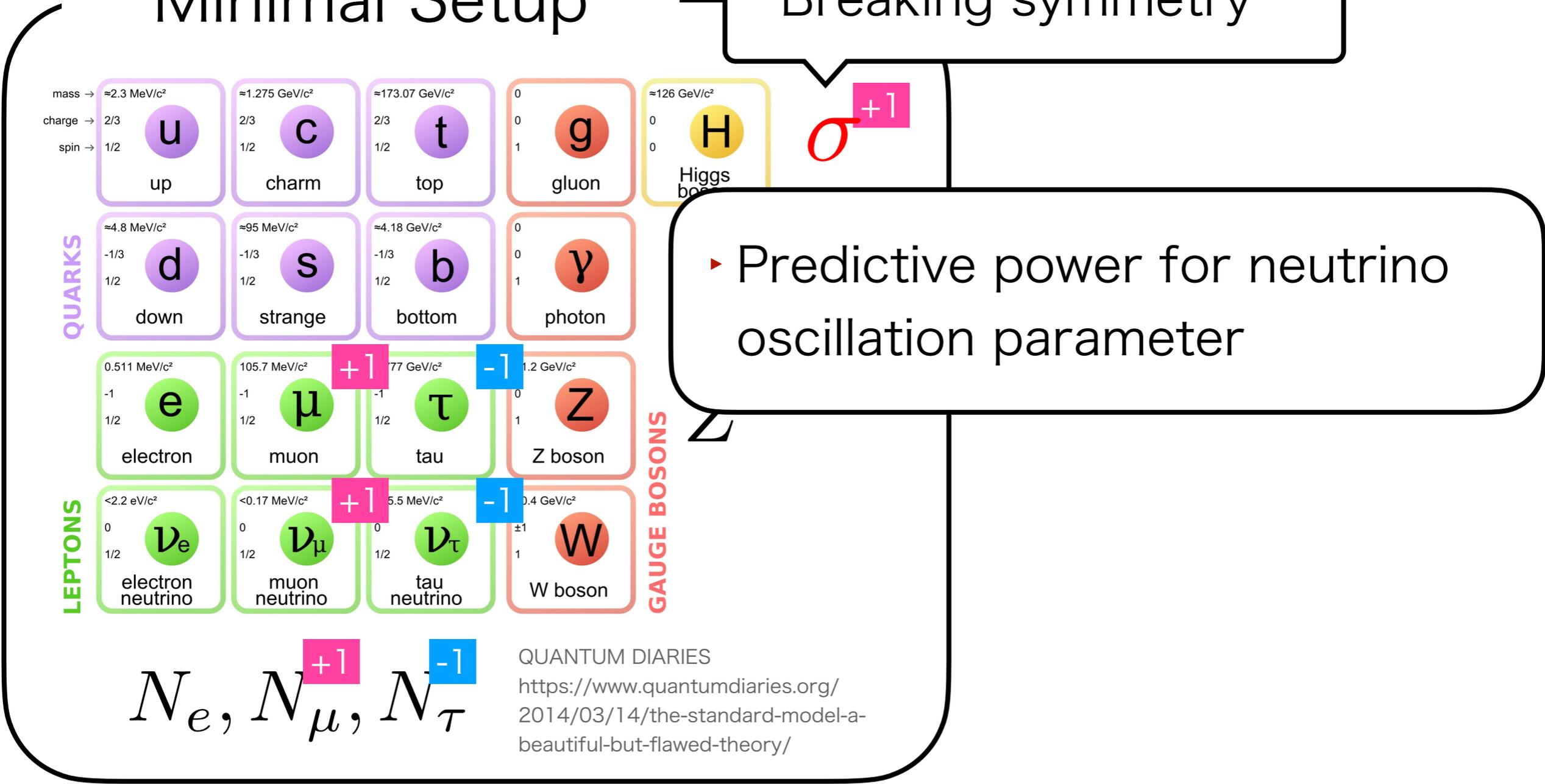
K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763

K. Asai, et.al., Phys.Rev.D 99 (2019) 5, 055029

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

## Minimal Setup

SM singlet  
Breaking symmetry



K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763

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# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

## Minimal Setup

SM singlet  
Breaking symmetry

mass →	≈2.3 MeV/c <sup>2</sup>	≈1.275 GeV/c <sup>2</sup>	≈173.07 GeV/c <sup>2</sup>	0	≈126 GeV/c <sup>2</sup>	
charge →	2/3	2/3	2/3	0	0	0
spin →	1/2	1/2	1/2	1	0	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson	$\sigma^{+1}$
<b>QUARKS</b>						
	≈4.8 MeV/c <sup>2</sup>	≈95 MeV/c <sup>2</sup>	≈4.18 GeV/c <sup>2</sup>	0		
	-1/3	-1/3	-1/3	0		
	1/2	1/2	1/2	1		
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b>γ</b> photon		
	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	91.1876 GeV/c <sup>2</sup>		
	-1	-1	-1	0		
	1/2	1/2	1/2	1		
	<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>Z</b> Z boson		
<b>LEPTONS</b>						
	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	5.5 MeV/c <sup>2</sup>	80.379 GeV/c <sup>2</sup>		
	0	0	0	±1		
	1/2	1/2	1/2	1		
	<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>W</b> W boson		

► Predictive power for neutrino oscillation parameter

► We evaluate BAO in this model with density matrix equation

A. Granelli, K. Hamaguchi, N. Nagata, M E. Ramirez-Quezada, and JW, JHEP 09 (2023) 079 [hep-ph 2305.18100]

$$N_e, N_\mu^{+1}, N_\tau^{-1}$$

QUANTUM DIA  
<https://www.quantumdiary.com/2014/03/14/the-beautiful-but-flawed>

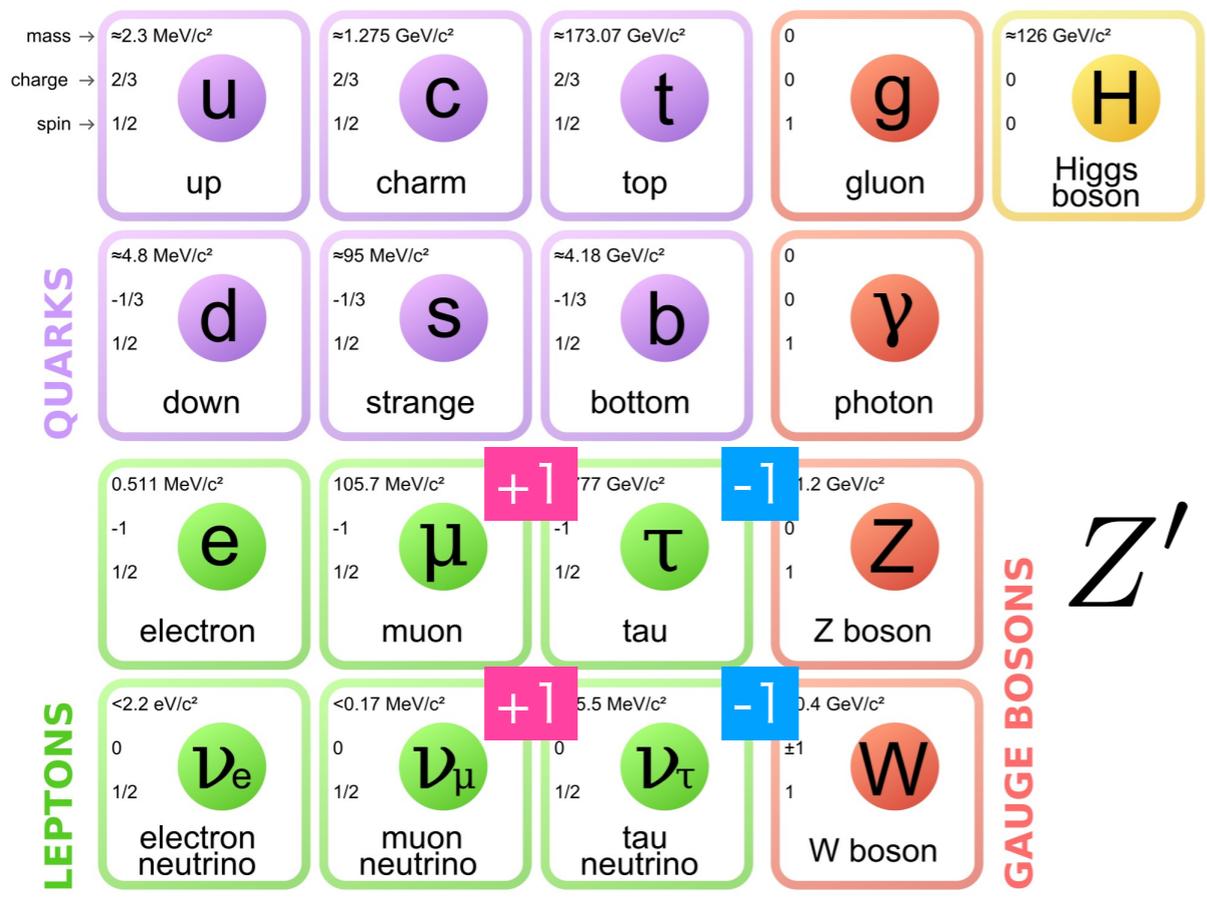
K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763  
K. Asai, et.al., Phys.Rev.D 99 (2019) 5, 055029

# Outline

- ✓ Introduction
- ▶ Minimal Gauged  $U(1)_{L_\mu - L_\tau}$  Model
- ▶ Thermal LG in  $U(1)_{L_\mu - L_\tau}$  model
- ▶ Flavor effect & DME
- ▶ Result
- ▶ Summary

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

## Minimal Setup



$\sigma$  **+1**

$\langle \sigma \rangle \gg 10^{10}$  GeV  
Interacting with Sterile neutrino

$Z'$

$N_e, N_\mu$  **+1**,  $N_\tau$  **-1**

QUANTUM DIARIES  
<https://www.quantumdiaries.org/2014/03/14/the-standard-model-a-beautiful-but-flawed-theory/>

K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763  
K. Asai, et.al., Phys.Rev.D 99 (2019) 5, 055029

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

$$\Delta\mathcal{L} = -\lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\ - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \lambda_{e\mu} \sigma N_e^c N_\mu^c - \lambda_{e\tau} \sigma^* N_e^c N_\tau^c + h.c.$$

After H and  $\sigma$  getting VEVs...

$$\mathcal{L}_{mass} = -(\nu_e, \nu_\mu, \nu_\tau) \mathcal{M}_D \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} - \frac{1}{2} (N_e^c, N_\mu^c, N_\tau^c) \mathcal{M}_R \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} + h.c.$$

Where

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \quad \mathcal{M}_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu} \langle \sigma \rangle & \lambda_{e\tau} \langle \sigma \rangle \\ \lambda_{e\mu} \langle \sigma \rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau} \langle \sigma \rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry

Because of this symmetry, structure of both Dirac and Majorana mass terms are tightly restricted.

→ Strong predictive power for the neutrino sector

$$\mathcal{M}_{\nu L} \simeq -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

$$U_{PMNS}^T \mathcal{M}_{\nu L} U_{PMNS} = \text{diag}(m_1, m_2, m_3)$$

Input

$$\Delta m^2, \delta m^2,$$

$$\theta_{12}, \theta_{23}, \theta_{31}$$

Output

$$\delta, \alpha_1, \alpha_2,$$

$$m_1, m_2, m_3$$

Where

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \quad \mathcal{M}_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu} \langle \sigma \rangle & \lambda_{e\tau} \langle \sigma \rangle \\ \lambda_{e\mu} \langle \sigma \rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau} \langle \sigma \rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

# $U(1)_{T_{12} - T_{13}}$ gauge symmetry

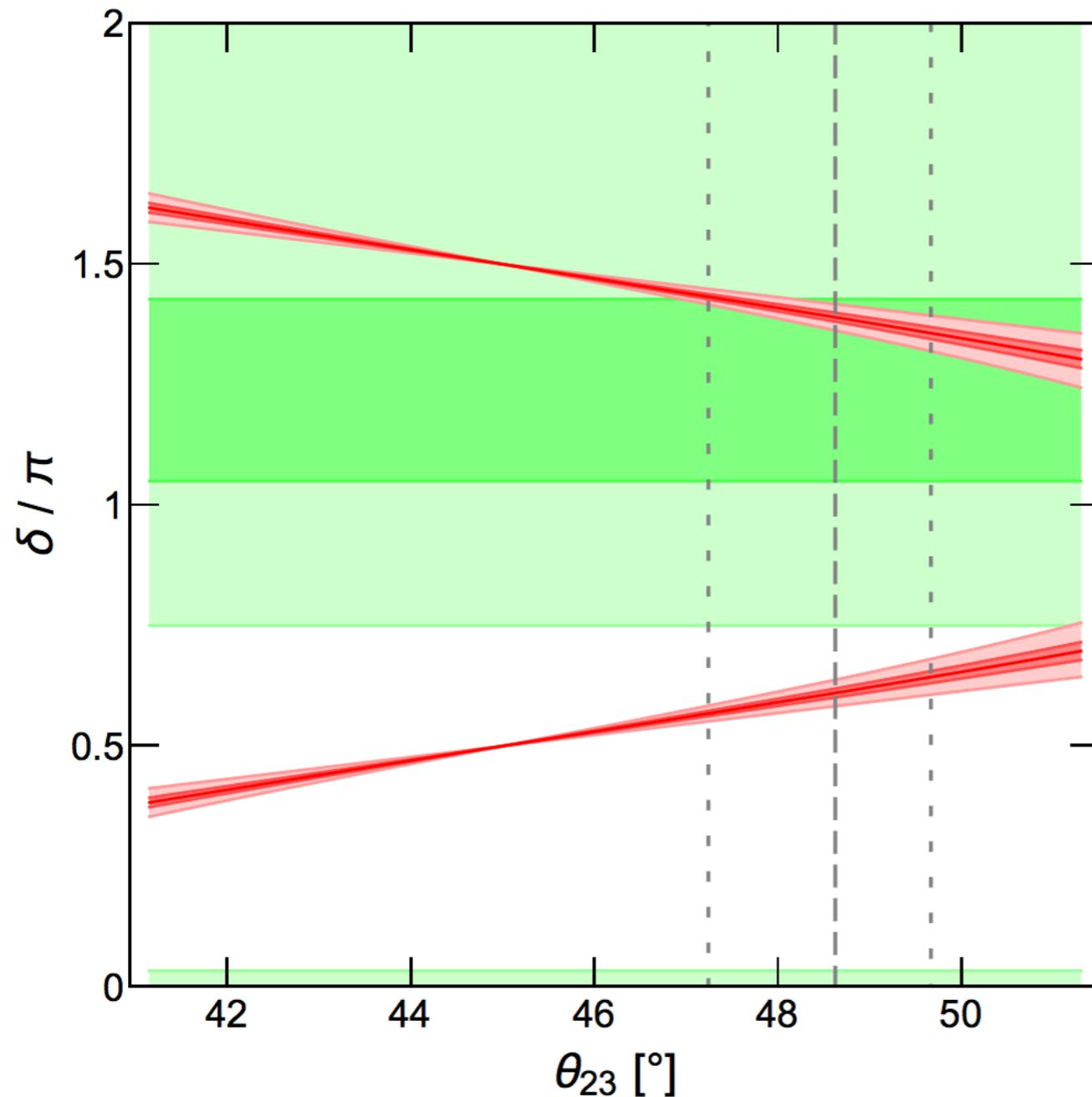


Fig taken from K. Asai et.al., JCAP 11 (2020) 013

K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763

K. Asai, et.al., Phys.Rev.D 99 (2019) 5, 055029

structure of both Dirac and  
slightly restricted.

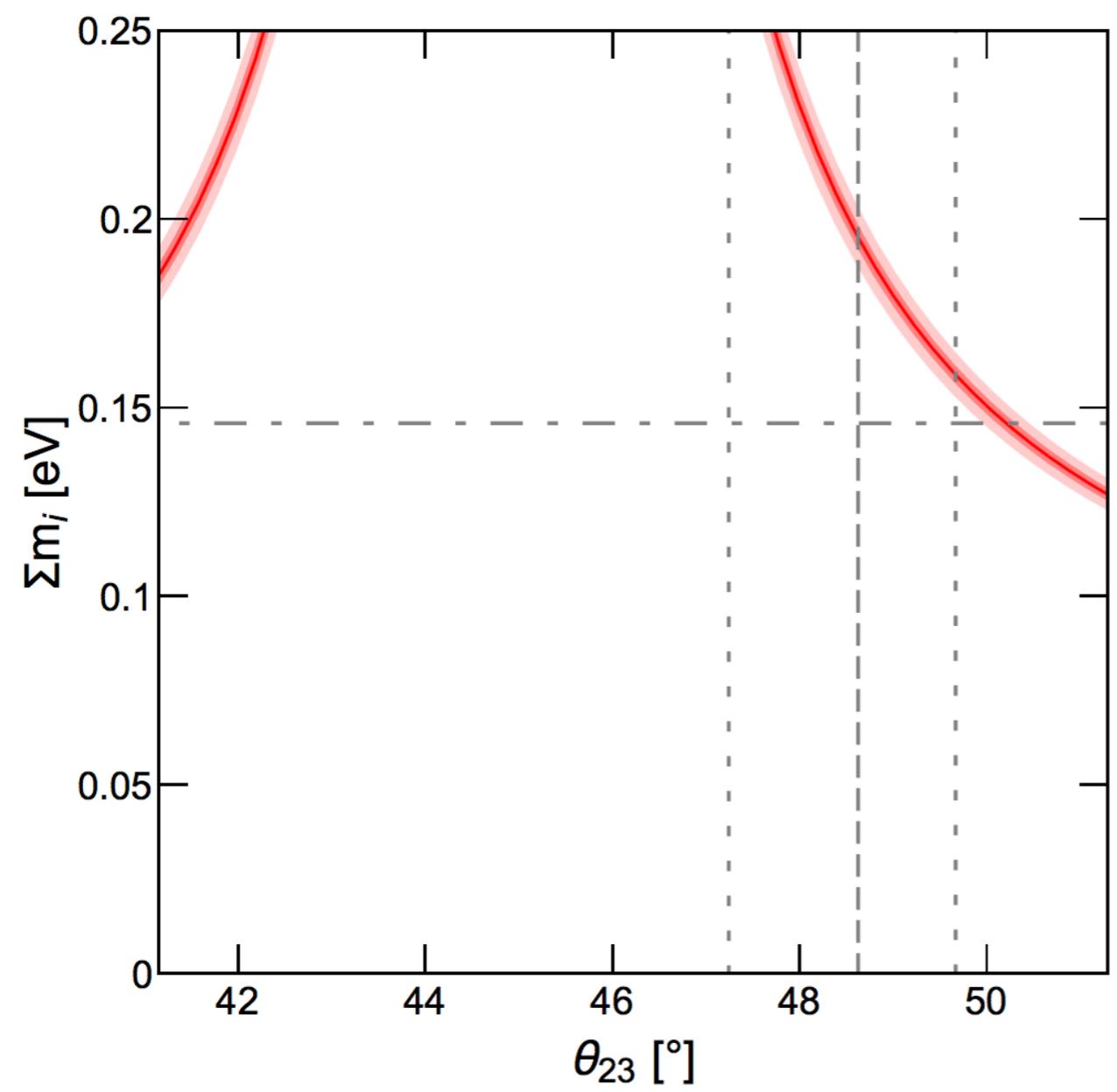
for the neutrino sector

Input  $\Delta m^2, \delta m^2,$   
 $\theta_{12}, \theta_{23}, \theta_{31}$   $\rightarrow$  Output  $\delta, \alpha_1, \alpha_2,$   
 $m_1, m_2, m_3$

$$\cos \delta \simeq \frac{\cot \theta_{12} \cot \theta_{23}}{\sin \theta_{13}}$$

Two solutions  $\delta, 2\pi - \delta$

# $U(1)_{L_\mu - L_\tau}$ gauge symmetry



structure of both Dirac and Majorana phases is tightly restricted.

• for the neutrino sector

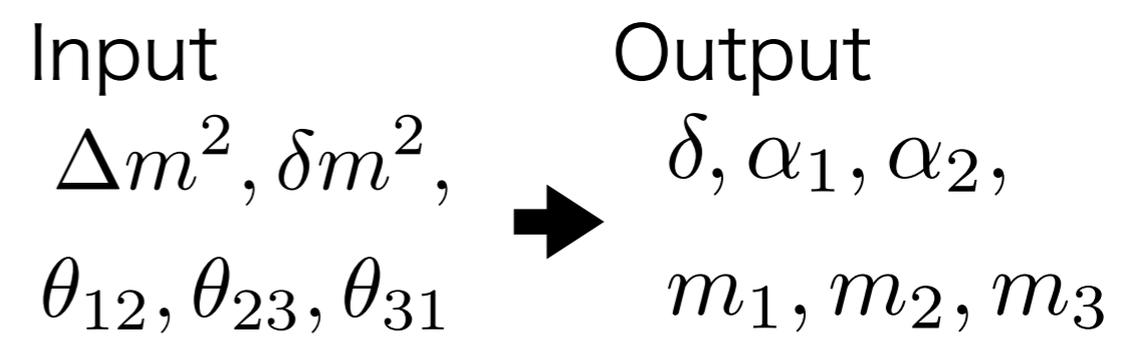


Fig taken from K. Asai et.al., JCAP 11 (2020) 013

K. Asai, et.al., Eur. Phys. J.C 77 (2017) 11, 763

K. Asai, et.al., Phys.Rev.D 99 (2019) 5, 055029

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# Thermal LG in $U(1)_{L_\mu - L_\tau}$ model <sup>12</sup>

To evaluate baryon asymmetry,

Input	Output	
$\Delta m^2, \delta m^2,$ $\theta_{12}, \theta_{23}, \theta_{31}$	$\delta, \alpha_1, \alpha_2,$ $m_1, m_2, m_3$	$\Rightarrow \mathcal{M}_{\nu L} = U_{PMNS}^* \text{diag}(m_1, m_2, m_3) U_{PMNS}^{-1}$

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \Rightarrow \mathcal{M}_R \simeq -\mathcal{M}_D^T \mathcal{M}_{\nu L}^{-1} \mathcal{M}_D$$

$$\mathcal{M}_D, \mathcal{M}_R \Rightarrow \eta_b \quad \text{baryon asymmetry}$$

# Thermal LG in $U(1)_{L_\mu - L_\tau}$ model <sup>14</sup>

$$M_i \simeq 6 \times 10^{14} \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_1} \right) \lambda^2 \beta_i(\theta, \phi)$$

$$(\lambda_e, \lambda_\mu, \lambda_\tau) = \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$

$n_3) U_{PMNS}^{-1}$

$\nu_{12}, \nu_{23}, \nu_{31}$        $\nu_{\sigma 1}, \nu_{\sigma 2}, \nu_{\sigma 3}$

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \rightarrow \mathcal{M}_R \simeq -\mathcal{M}_D^T \mathcal{M}_{\nu_L}^{-1} \mathcal{M}_D$$

$\mathcal{M}_D, \mathcal{M}_R \rightarrow \eta_b$     baryon asymmetry

# Thermal LG in $U(1)_{L_\mu - L_\tau}$ model <sup>15</sup>

$$M_i \simeq 6 \times 10^{14} \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_1} \right) \lambda^2 \beta_i(\theta, \phi)$$

Thermal LG works when

$$10^{11-12} \text{ GeV} \lesssim M_1$$

$n_3) U_{PMNS}^{-1}$

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \rightarrow \mathcal{M}_R \simeq -\mathcal{M}_D^T \mathcal{M}_{\nu_L}^{-1} \mathcal{M}_D$$

$\mathcal{M}_D, \mathcal{M}_R \rightarrow \eta_b$  baryon asymmetry

# Thermal LG in $U(1)_{L_\mu - L_\tau}$ model <sup>16</sup>

$$M_i \simeq 6 \times 10^{14} \text{ GeV} \left( \frac{0.05 \text{ eV}}{\lambda^2 \beta_i(\theta, \phi)} \right)$$

Thermal LG works when

$$10^{11-12} \text{ GeV} \lesssim M_1$$

$y_\tau$  in thermal equilibrium at

$$T \sim 10^{12} \text{ GeV}$$

Flavor effect affects thermal LG

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix}$$

R. Barbieri, et.al., Nucl.Phys.B 575 (2000) 61-77

E. Nardi, et.al., JHEP 01 (2006) 164

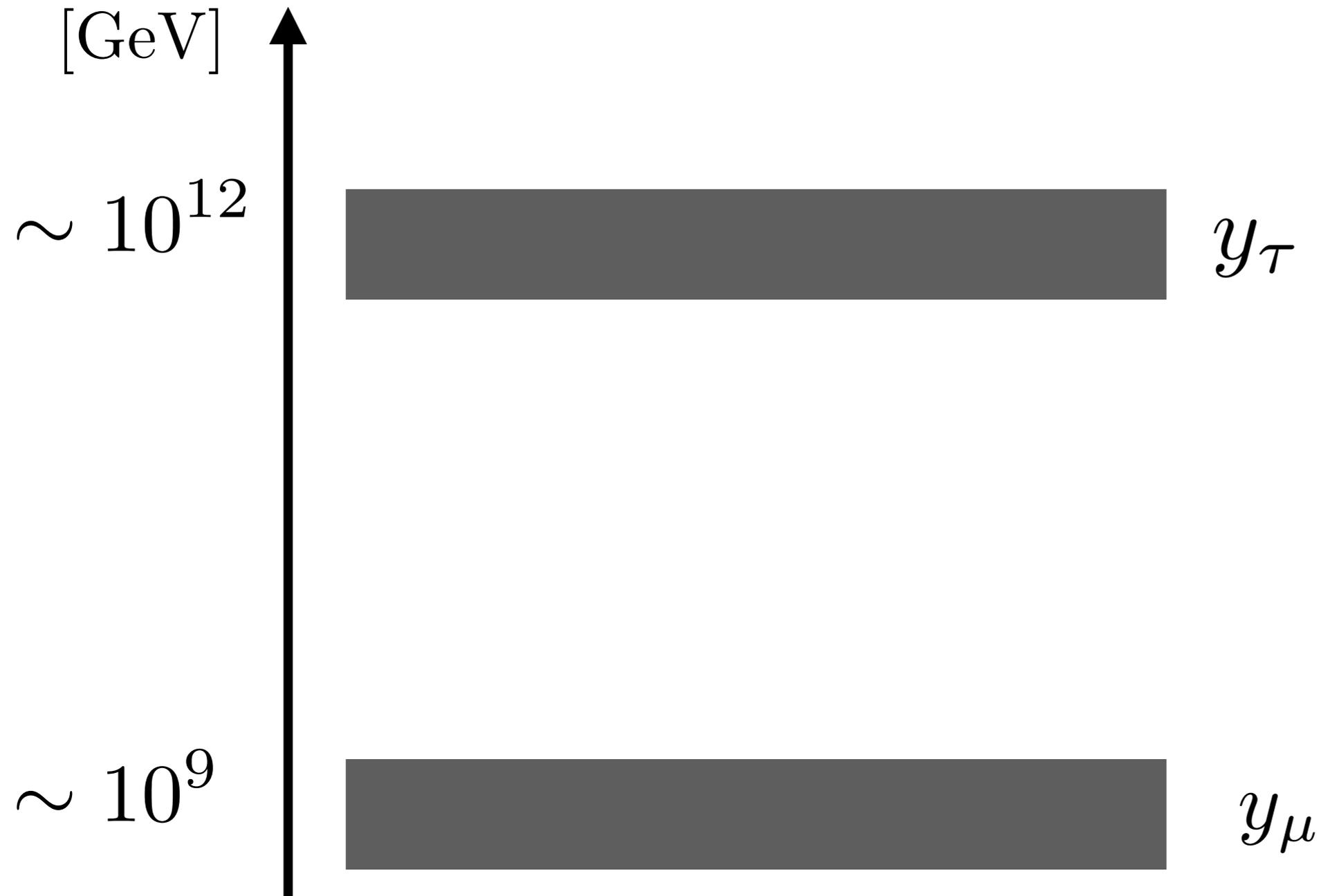
A. Abada, et.al., JCAP 04 (2006) 004

$\mathcal{M}_D, \mathcal{M}_R \rightarrow \eta_b$  baryon asymmetry

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# Flavor effect on LG

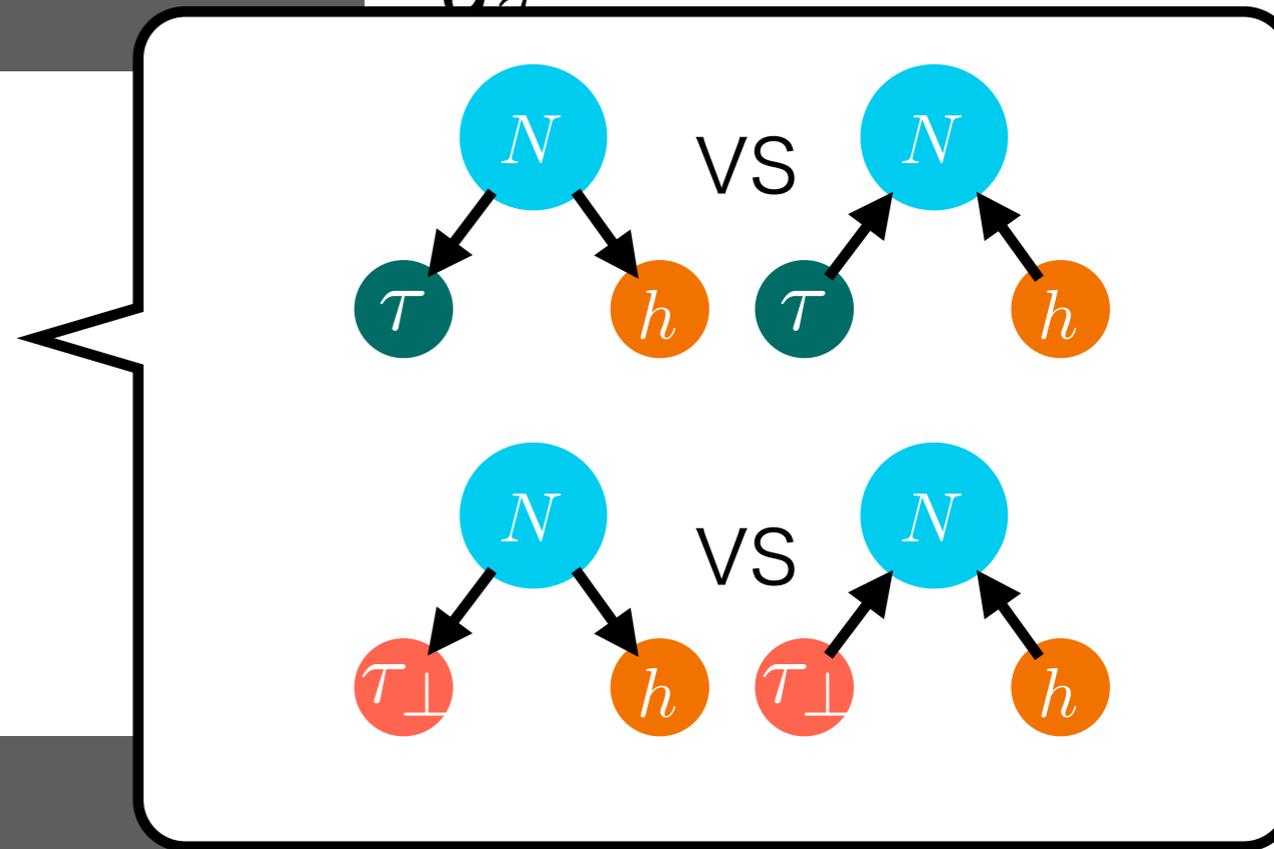
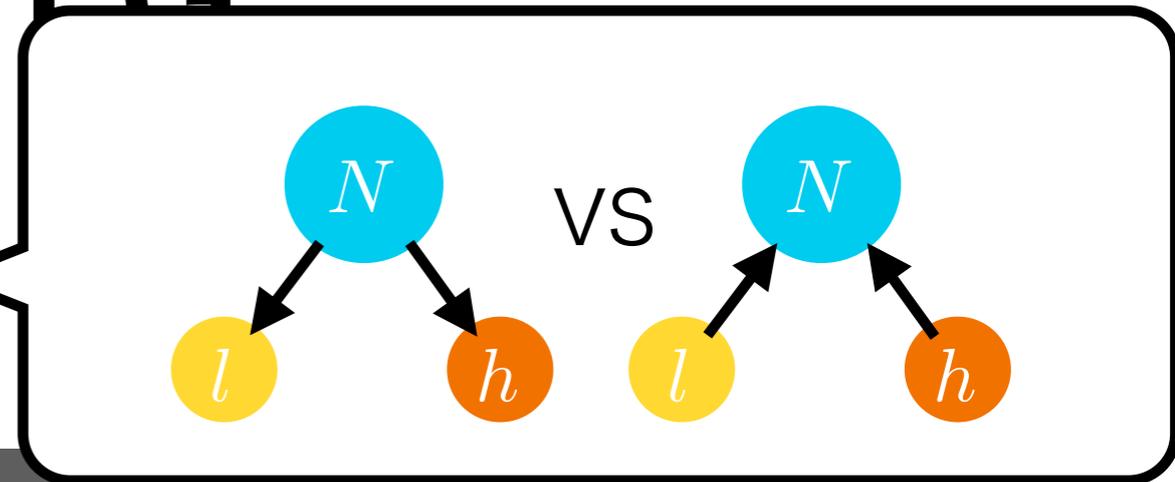
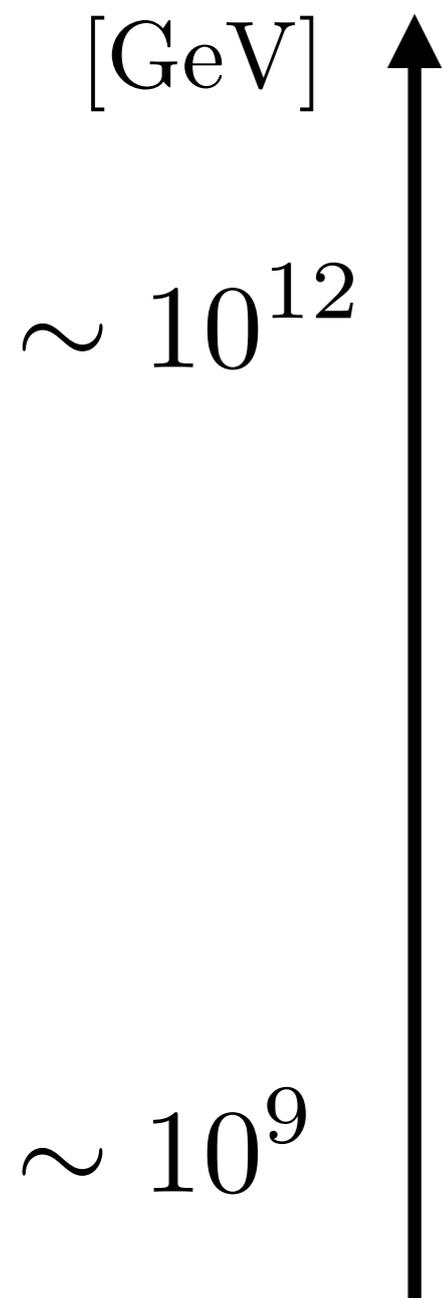


R. Barbieri, et.al., Nucl.Phys.B 575 (2000) 61-77

E. Nardi, et.al., JHEP 01 (2006) 164

A. Abada, et.al., JCAP 04 (2006) 004

# Flavor effect on $U_1$



R. Barbieri, et.al., Nucl.Phys.B 575 (2000) 61-77

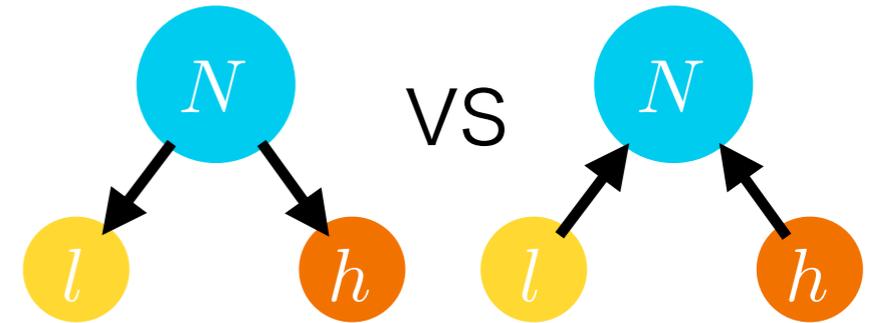
E. Nardi, et.al., JHEP 01 (2006) 164

A. Abada, et.al., JCAP 04 (2006) 004

$$\epsilon_1 \simeq -\frac{3M_1}{8\pi v^2} \frac{\sum_i m_i^2 \text{Im} [(R_{1i})^2]}{\sum_i m_i |R_{1i}|^2}$$

S. Davidson and A. Ibarra, Phys.Lett.B 535 (2002) 25-32

$\sim 10^{12}$



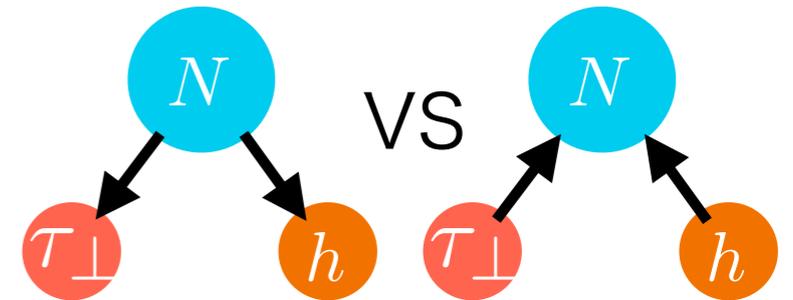
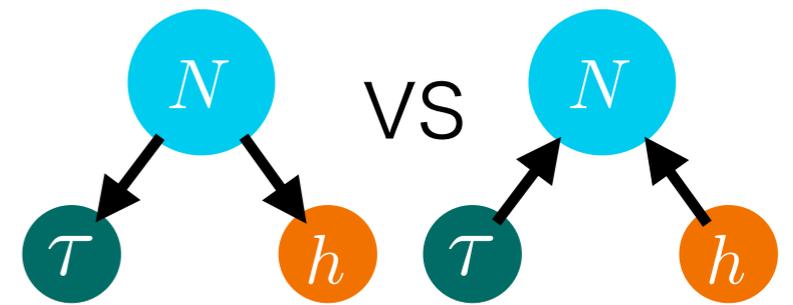
$U_\tau$

$$\epsilon_{1,\alpha} \propto \frac{\text{Im} \left[ \sum_{jk} m_j^{1/2} m_k^{1/2} U_{\alpha j}^* U_{\alpha k} R_{1j} R_{1k} \right]}{\sum_i m_i |R_{1i}|^2}$$

E. Nardi, et.al., JHEP 01 (2006) 164

E. Molinaro and S. T. Petcov, Eur. Phys. J. C 61 (2009) 93-109

$\sim 10^9$



$U_\mu$

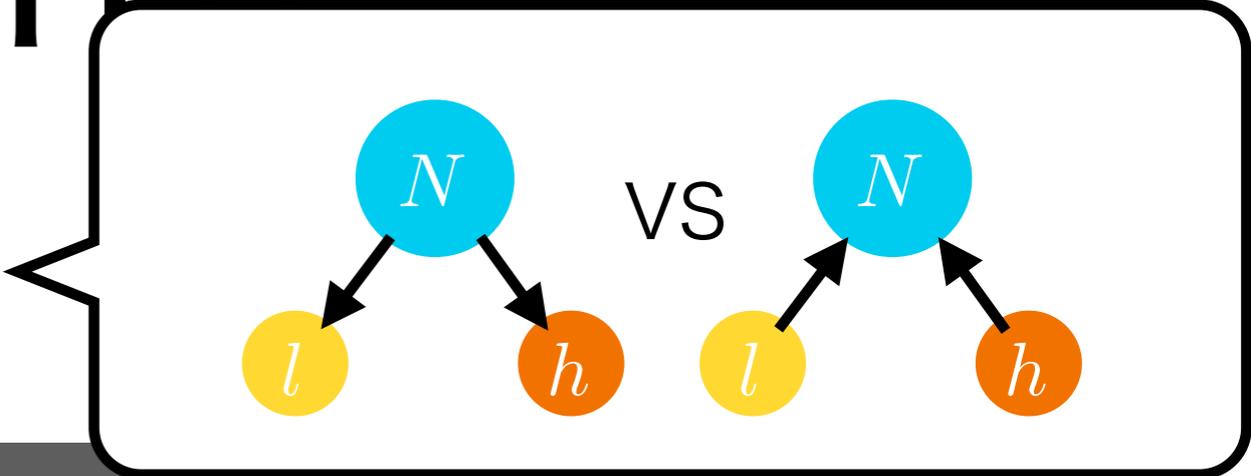
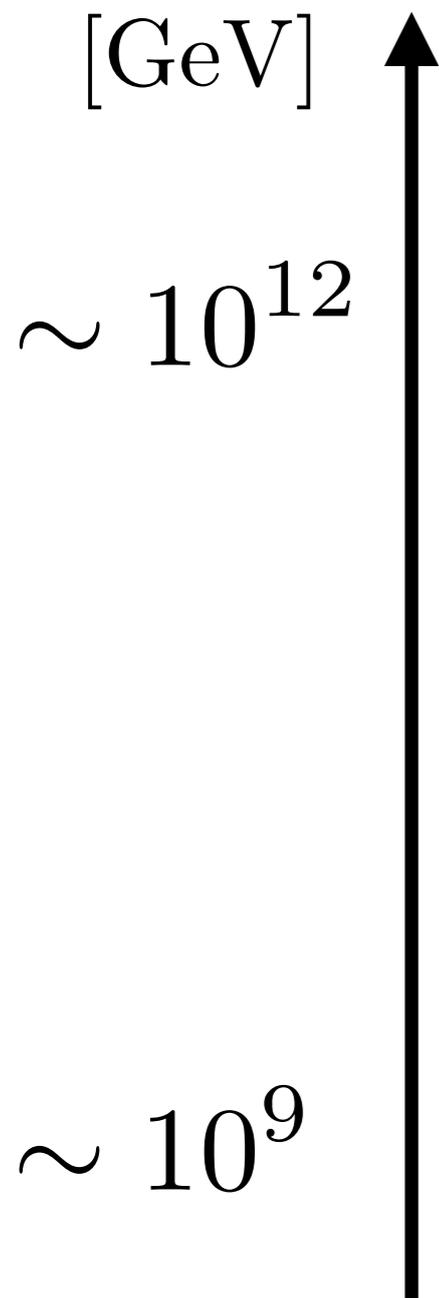
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E. Nardi, et.al., JHEP 01 (2006) 164

A. Abada, et.al., JCAP 04 (2006) 004

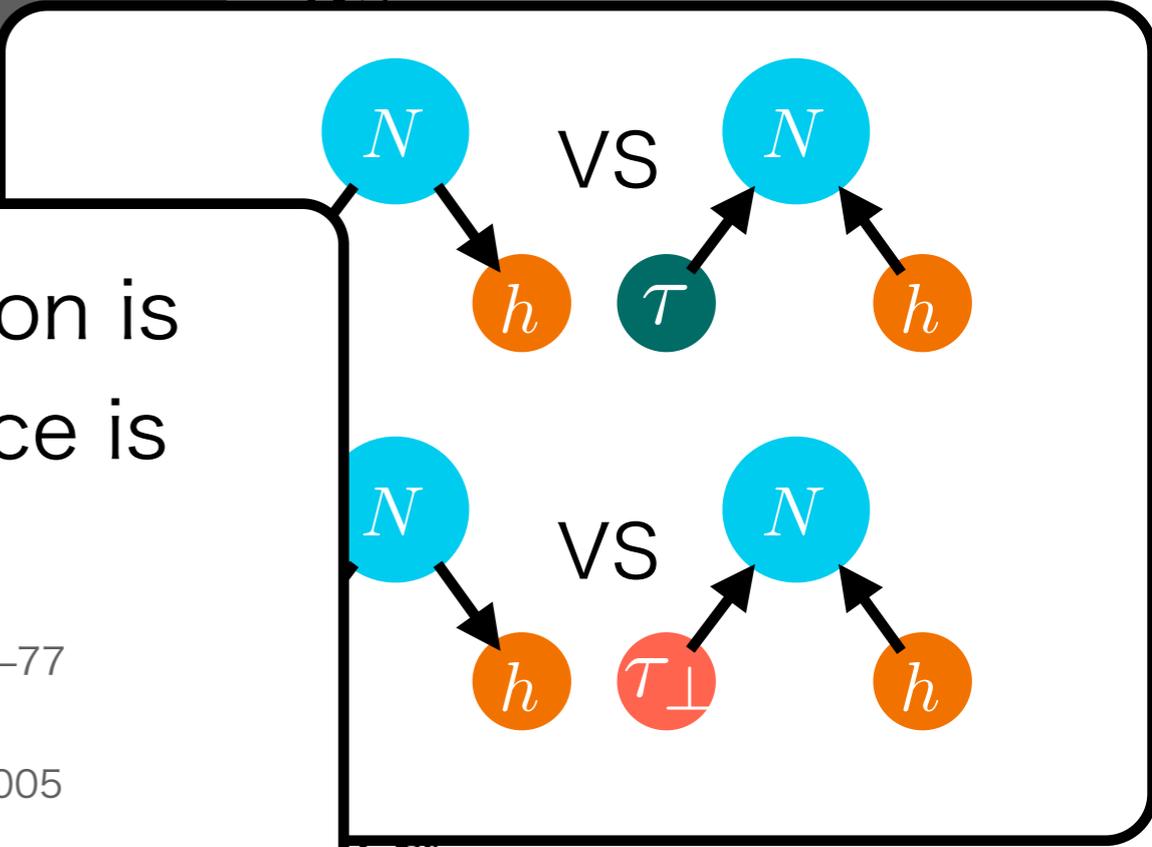
Dependence PMNS phase is important.

# Flavor effect on LG



Density Matrix Equation is required. (Decoherence is important)

R. Barbieri, et.al., Nucl. Phys. B 575 (2000) 61-77  
 A. Abada, et.al., JCAP 04 (2006) 004  
 A. De Simone and A. Riotto, JCAP 02 (2007) 005  
 S. Blanchet, et.al., JCAP 01 (2013) 041



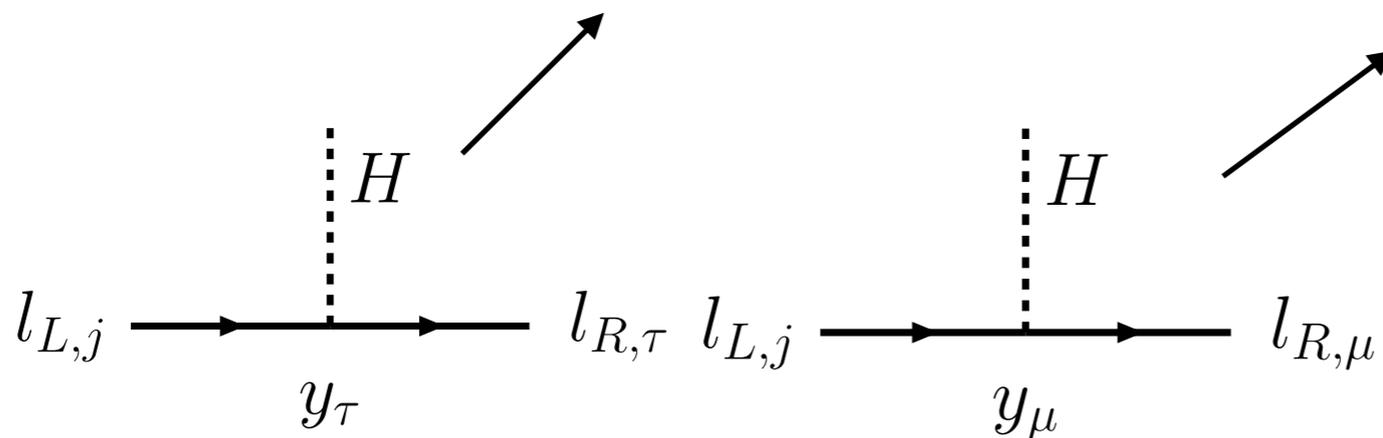
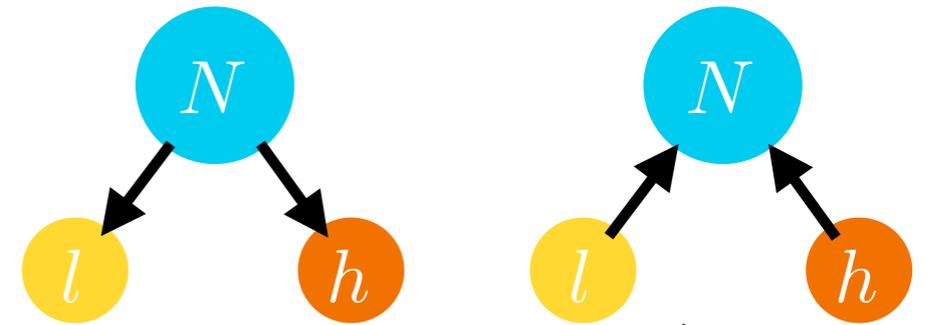
R. Barbieri, et.al., Nucl.Phys.B 575 (2000) 61-77  
 E. Nardi, et.al., JHEP 01 (2006) 164  
 A. Abada, et.al., JCAP 04 (2006) 004

# Density Matrix Equation

$$\frac{dN_{N_j}}{dz} = -D_j(N_{N_j} - N_{N_j}^{\text{eq}})$$

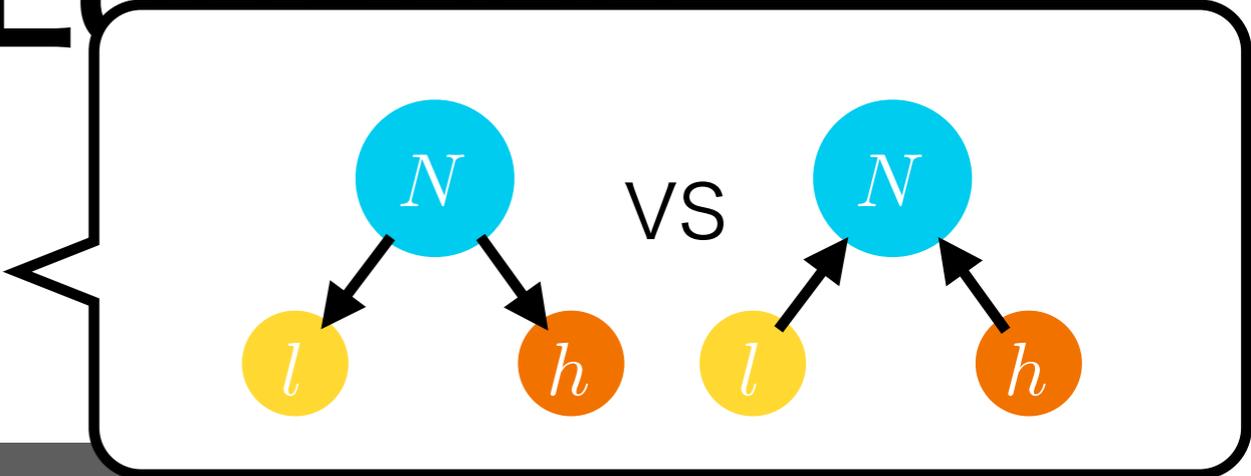
$$\frac{dN_{\alpha\beta}}{dz} = \sum_{j=1}^3 \left[ \epsilon_{\alpha\beta}^{(j)} D_j(N_{N_j} - N_{N_j}^{\text{eq}}) - \frac{1}{2} W_j \left\{ P^{0(j)}, N \right\}_{\alpha\beta} \right]$$

$$- \frac{\Gamma_\tau}{Hz} [I_\tau, [I_\tau, N]]_{\alpha\beta} - \frac{\Gamma_\mu}{Hz} [I_\mu, [I_\mu, N]]_{\alpha\beta} ,$$



- R. Barbieri, et.al., Nucl. Phys. B 575 (2000) 61–77
- A. Abada, et.al., JCAP 04 (2006) 004
- A. De Simone and A. Riotto, JCAP 02 (2007) 005
- S. Blanchet, et.al., JCAP 01 (2013) 041

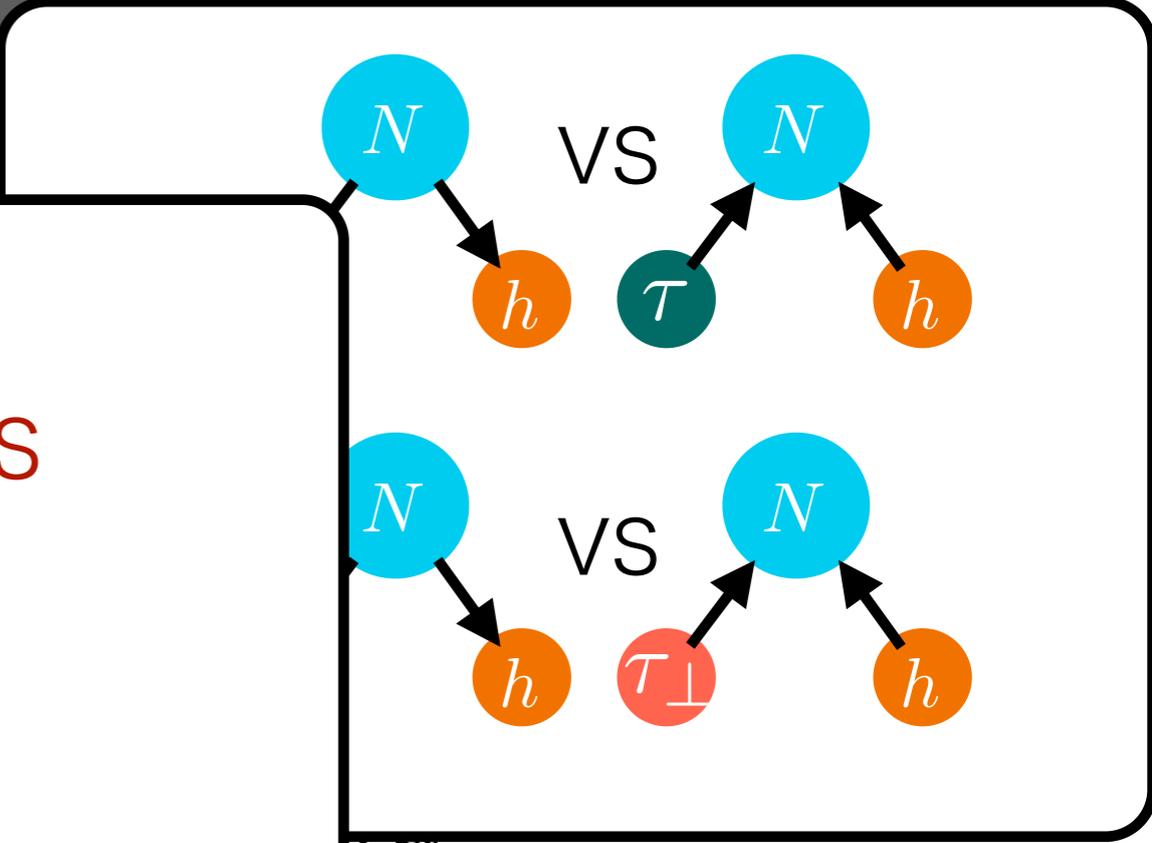
# Density Matrix Equation



Numerical calculation  
with DME by **ULYSSES**

A. Granelli, et.al., Comput.Phys.Commun.  
262 (2021) 107813

A. Granelli, et.al., Comput.Phys.Commun.  
291 (2023) 108834



et.al., Nucl. Phys. B 575 (2000) 61–77

A. Abada, et.al., JCAP 04 (2006) 004

A. De Simone and A. Riotto, JCAP 02 (2007) 005

S. Blanchet, et.al., JCAP 01 (2013) 041

# Assumption

- ▶  $U(1)_{L_\mu - L_\tau}$  gauge symmetry is never restored after the reheating
- ▶ singlet scalar field associated  $\sigma$  and  $Z'$  are sufficiently heavy so that these fields are always absent from the thermal bath

▶  $\langle \sigma \rangle \gg T_R$

- ▶ The masses of all three right-handed neutrinos are smaller than the reheating temperature.

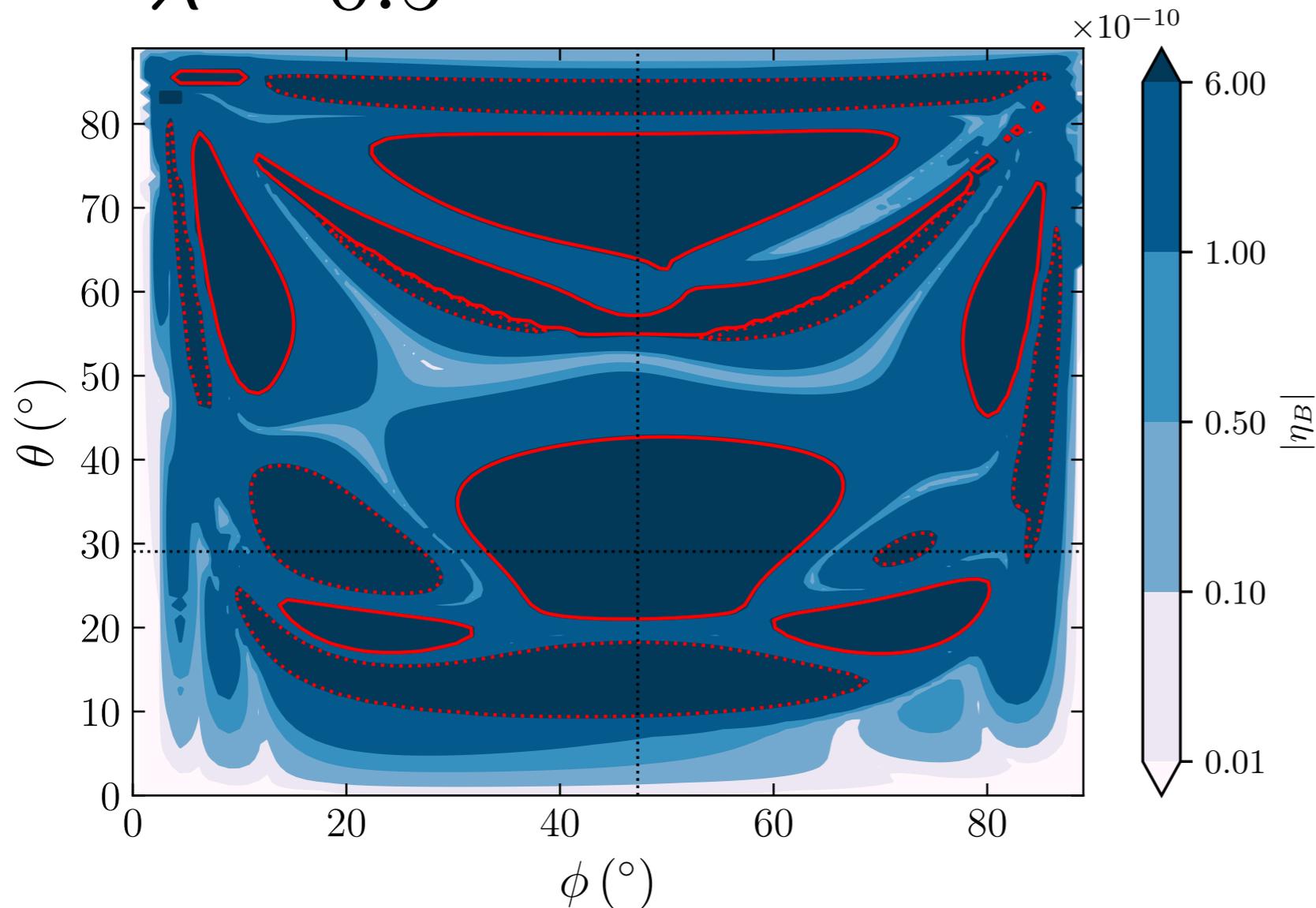
▶  $|M_{ee, \mu\tau}|, |\lambda_{e\mu, e\tau} \langle \sigma \rangle| < T_R$

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# Result

$$\lambda = 0.5$$



## Set I

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.58^\circ$$

$$\theta_{23} = 39.7^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 = 2.507 \times 10^{-3} \text{ eV}^2$$

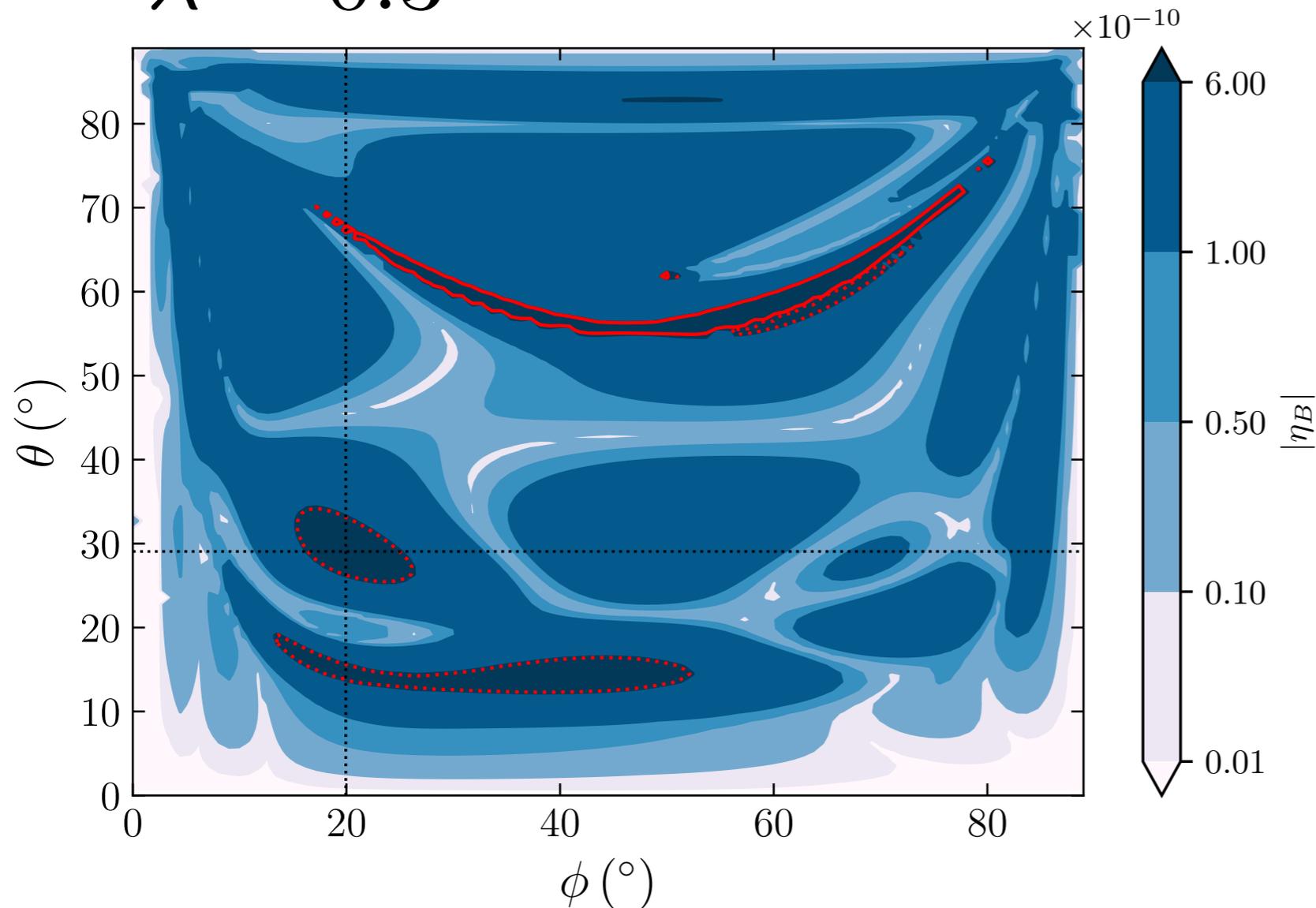
Input parameters are taken from NuFit ver 5.2

NuFIT Collaboration, NuFIT v5.2, <http://www.nu-fit.org>.

I. Esteban, et.al., JHEP 09 (2020) 178

# Result

$$\lambda = 0.3$$



## Set I

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.58^\circ$$

$$\theta_{23} = 39.7^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5} \text{ eV}^2$$

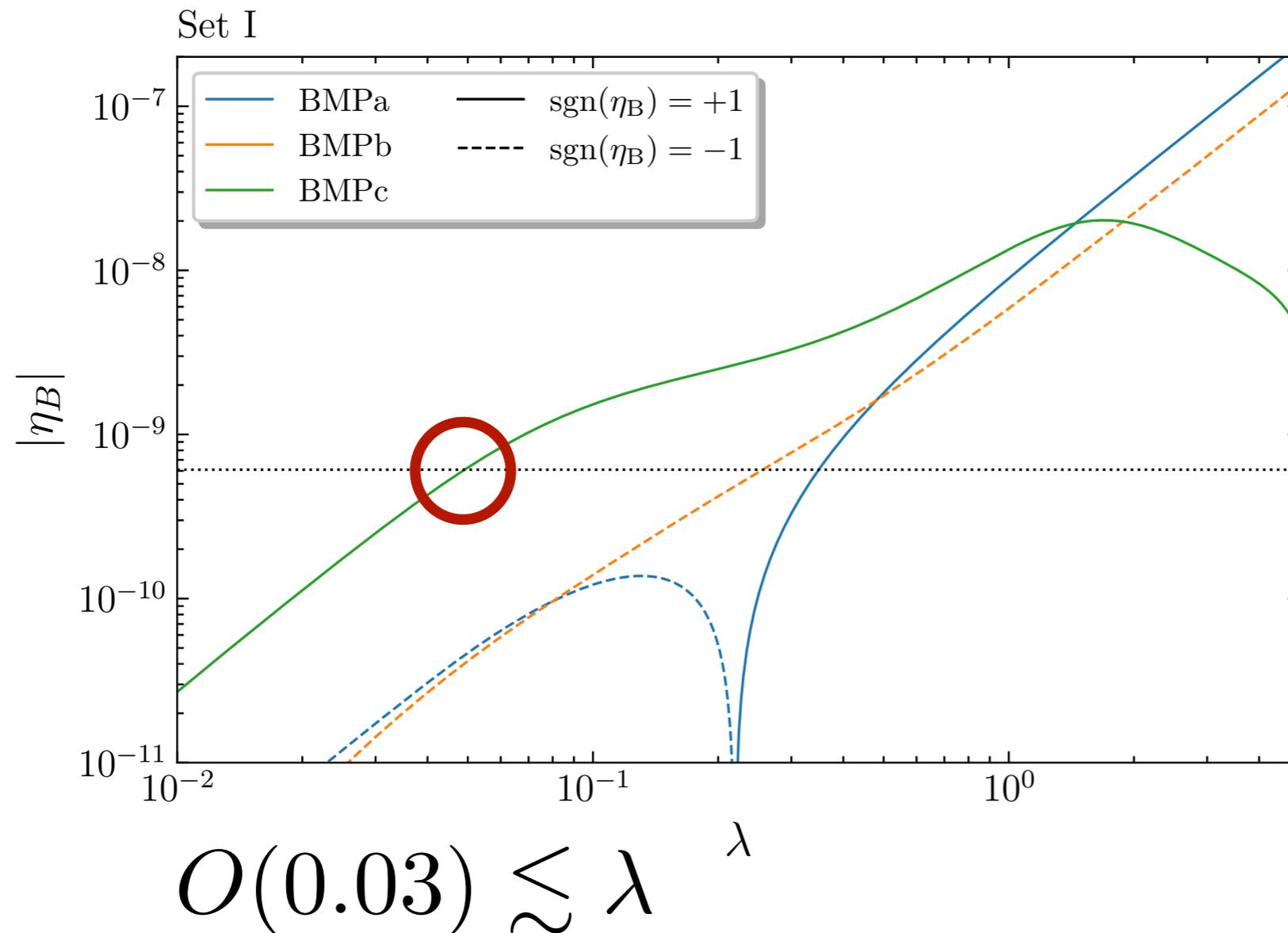
$$\Delta m_{31}^2 = 2.507 \times 10^{-3} \text{ eV}^2$$

Input parameters are taken from NuFit ver 5.2

NuFIT Collaboration, NuFIT v5.2, <http://www.nu-fit.org>.

I. Esteban, et.al., JHEP 09 (2020) 178

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A. Granelli, K. Hamaguchi, N. Nagata, M E. Ramirez-Quezada, and JW, JHEP 09 (2023) 079 [hep-ph 2305.18100]

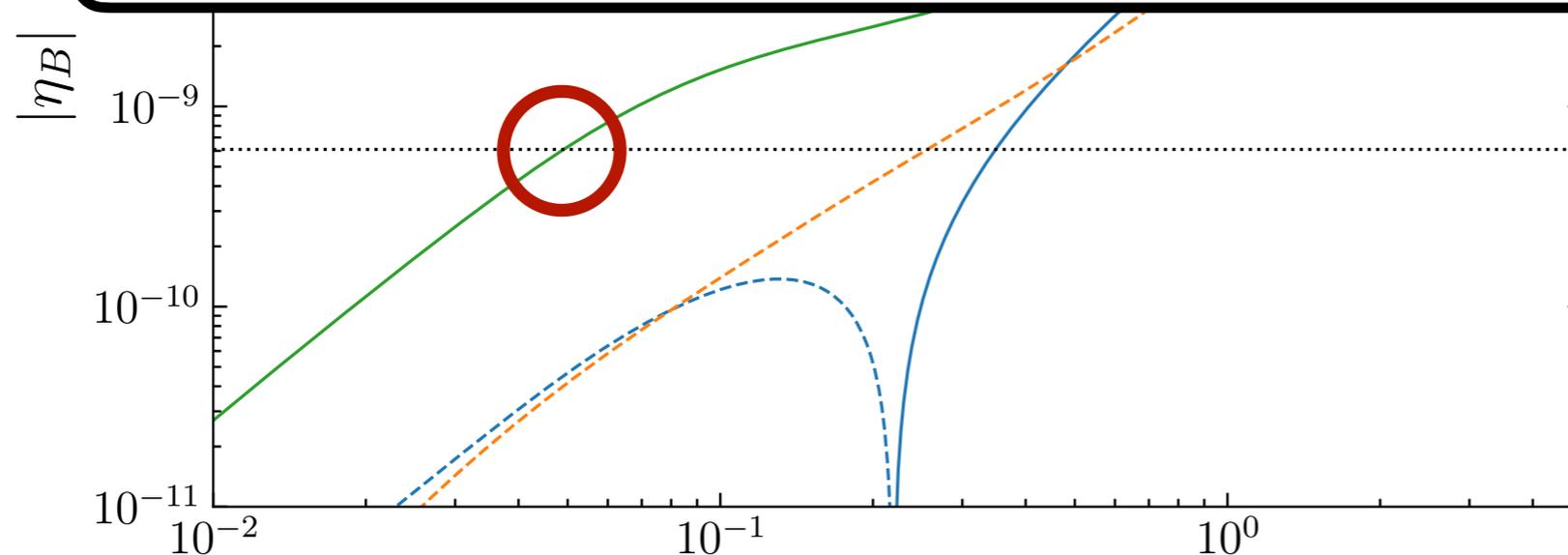
This is larger than the conventional bound due to the restriction from gauge symmetry.

S. Davidson, A. Ibarra, Phys. Lett. B 535 (2002) 25-32

# Result

$$M_i \simeq 6 \times 10^{14} \text{ GeV} \left( \frac{0.05 \text{ eV}}{m_1} \right) \lambda^2 \beta_i(\theta, \phi)$$

$$\blacktriangleright 10^{11-12} \text{ GeV} \lesssim M_1$$



$$O(0.03) \lesssim \lambda$$

## Set I

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.58^\circ$$

$$\theta_{23} = 39.7^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 = 2.507 \times 10^{-3} \text{ eV}^2$$

Input parameters are taken from NuFit ver 5.2

NuFIT Collaboration, NuFIT v5.2, <http://www.nu-fit.org>.

I. Esteban, et.al., JHEP 09 (2020) 178

A. Granelli, K. Hamaguchi, N. Nagata, M E. Ramirez-Quezada, and JW, JHEP 09 (2023) 079 [hep-ph 2305.18100]

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# Summary

- ▶ In Minimal gauged  $U(1)_{L_\mu-L_\tau}$  model, the phases and the sum of the light neutrino masses are predictable because of a restricted neutrino mass matrix structure.
- ▶ Additionally, in the context of thermal leptogenesis, the BAU can be computed in terms of the three remaining free variables
- ▶ Mass of the lightest RH  $\nu$ ,  $M_1 \gtrsim 10^{11-12}$  GeV setting LG scale in the considered model which is higher than Davidson-Ibarra bound.

**Backup**

# Benchmark Point

Set I

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.58^\circ$$

$$\theta_{23} = 39.7^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 = 2.507 \times 10^{-3} \text{ eV}^2$$

Set II

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.54^\circ$$

$$\theta_{23} = 51.9^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 = 2.511 \times 10^{-3} \text{ eV}^2$$

We have taken  $3\sigma$  ranges of the neutrino mixing angle  $\theta_{23}$  to avoid constraint on sum of neutrino mass.

Cf) NuFit data

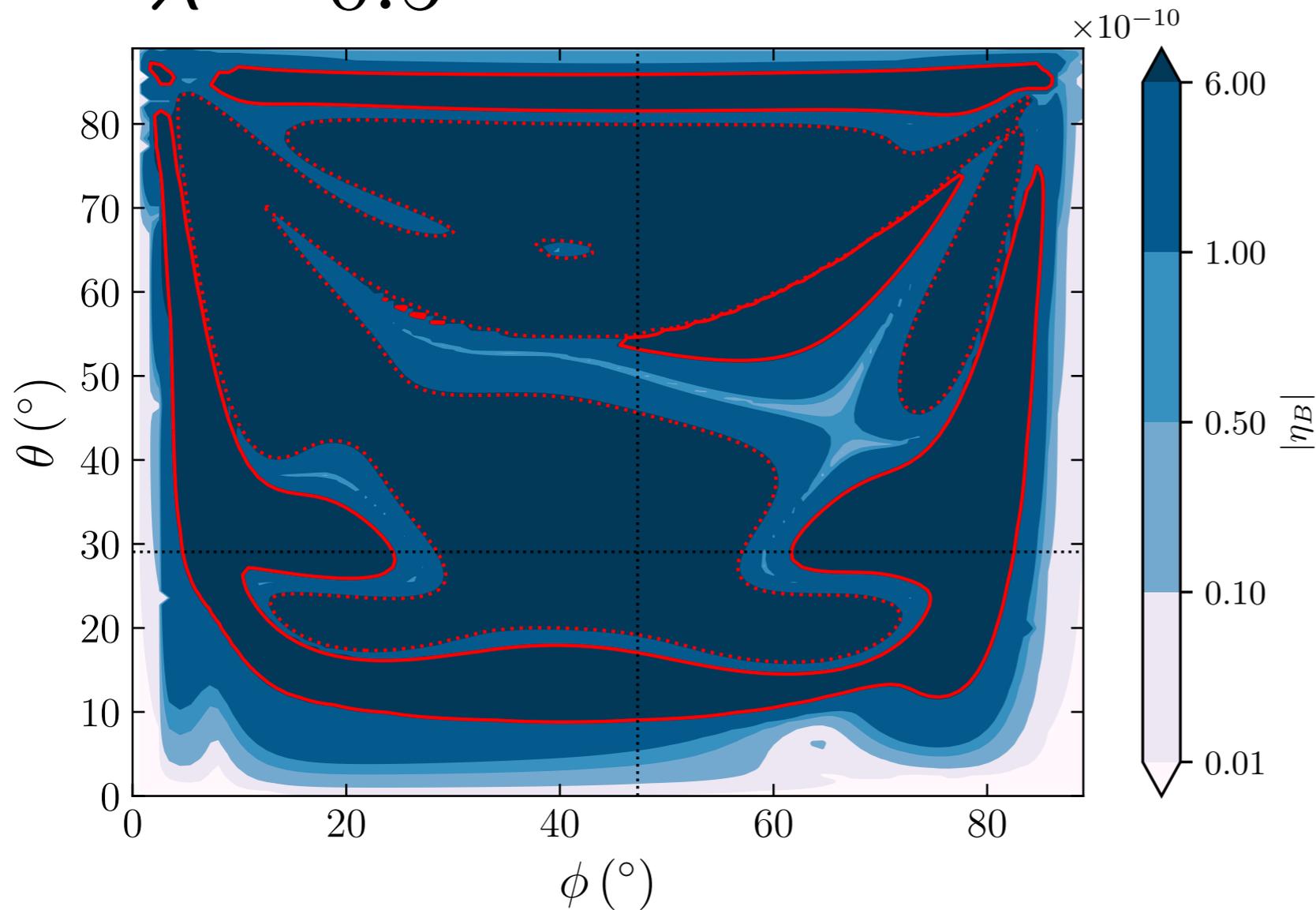
NuFIT Collaboration, NuFIT v5.2, <http://www.nu-fit.org>.

I. Esteban, et.al., JHEP 09 (2020) 178

Neutrino Masses and Mixing Parameters					
Parameters (units)	$\theta_{12}$ ( $^\circ$ )	$\theta_{13}$ ( $^\circ$ )	$\theta_{23}$ ( $^\circ$ )	$\Delta m_{21}^2$ ( $10^{-5} \text{ eV}^2$ )	$\Delta m_{31}^2$ ( $10^{-3} \text{ eV}^2$ )
With SK	$33.41^{+0.75}_{-0.72}$	$8.58^{+0.11}_{-0.11}$	$42.2^{+1.1}_{-0.9}$	$7.41^{+0.21}_{-0.20}$	$2.507^{+0.026}_{-0.027}$
$3\sigma$ range	[31.31, 35.74]	[8.23, 8.91]	[39.7, 51.0]	[6.82, 8.03]	[2.427, 2.590]
Without SK	$33.41^{+0.75}_{-0.72}$	$8.54^{+0.11}_{-0.12}$	$49.1^{+1.0}_{-1.3}$	$7.41^{+0.21}_{-0.20}$	$2.511^{+0.028}_{-0.027}$
$3\sigma$ range	[31.31, 35.74]	[8.19, 8.89]	[39.6, 51.9]	[6.82, 8.03]	[2.427, 2.590]

# Result

$$\lambda = 0.5$$



## Set II

$$\theta_{12} = 33.41^\circ$$

$$\theta_{13} = 8.54^\circ$$

$$\theta_{23} = 51.9^\circ$$

$$\Delta m_{21}^2 = 7.41 \times 10^{-5}$$

$$\Delta m_{31}^2 = 2.511 \times 10^{-3} \text{ eV}^2.$$

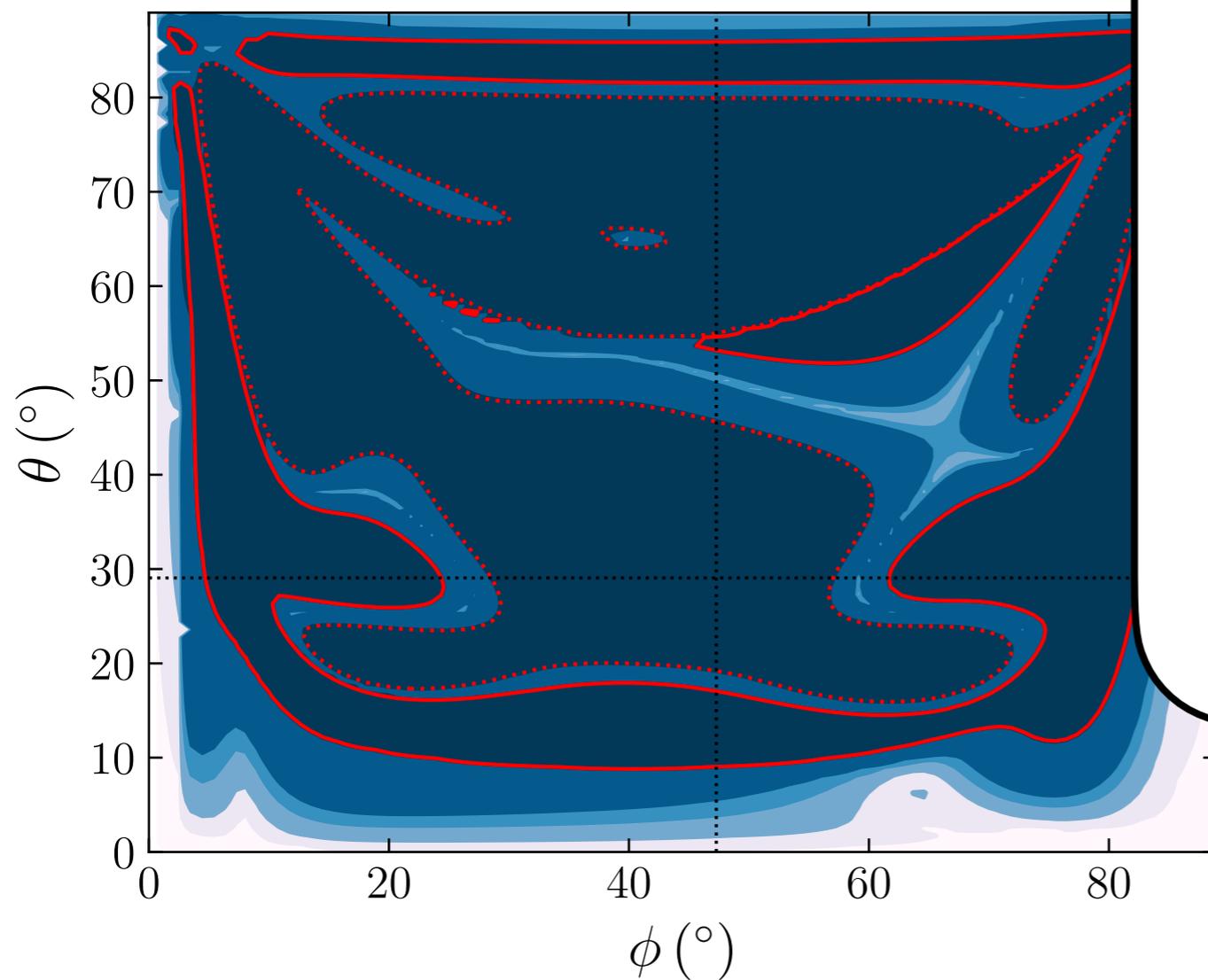
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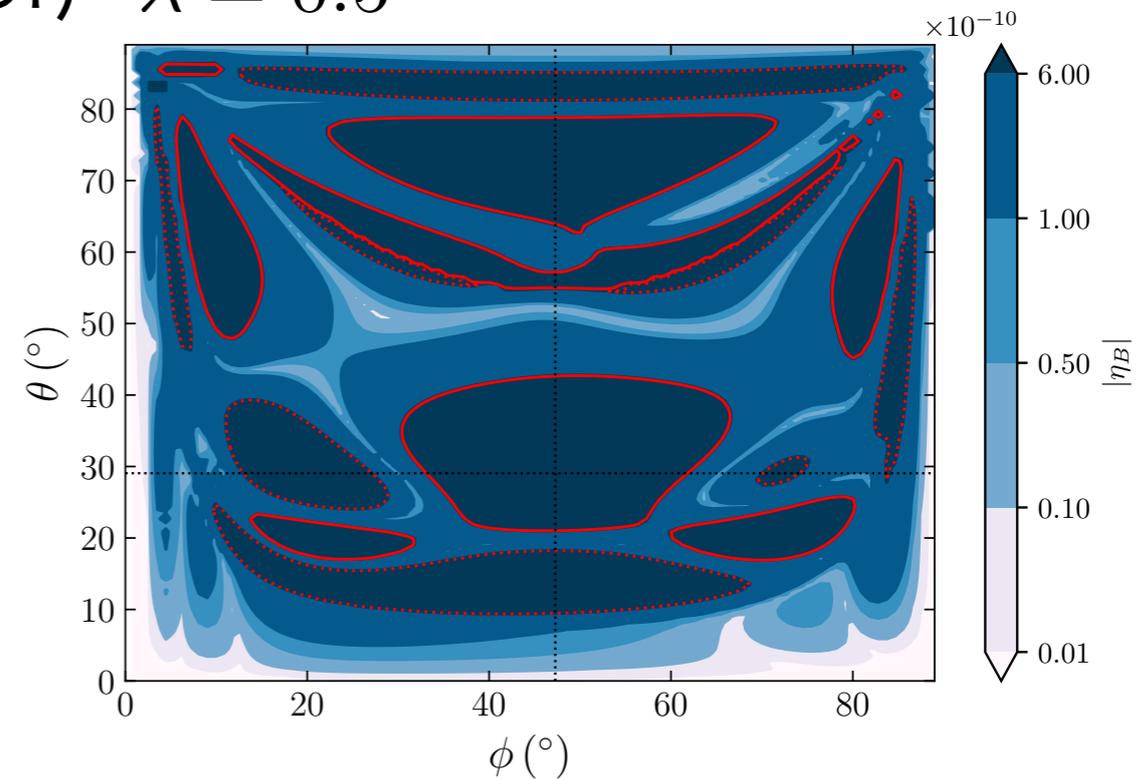
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# Result

$$\lambda = 0.5$$



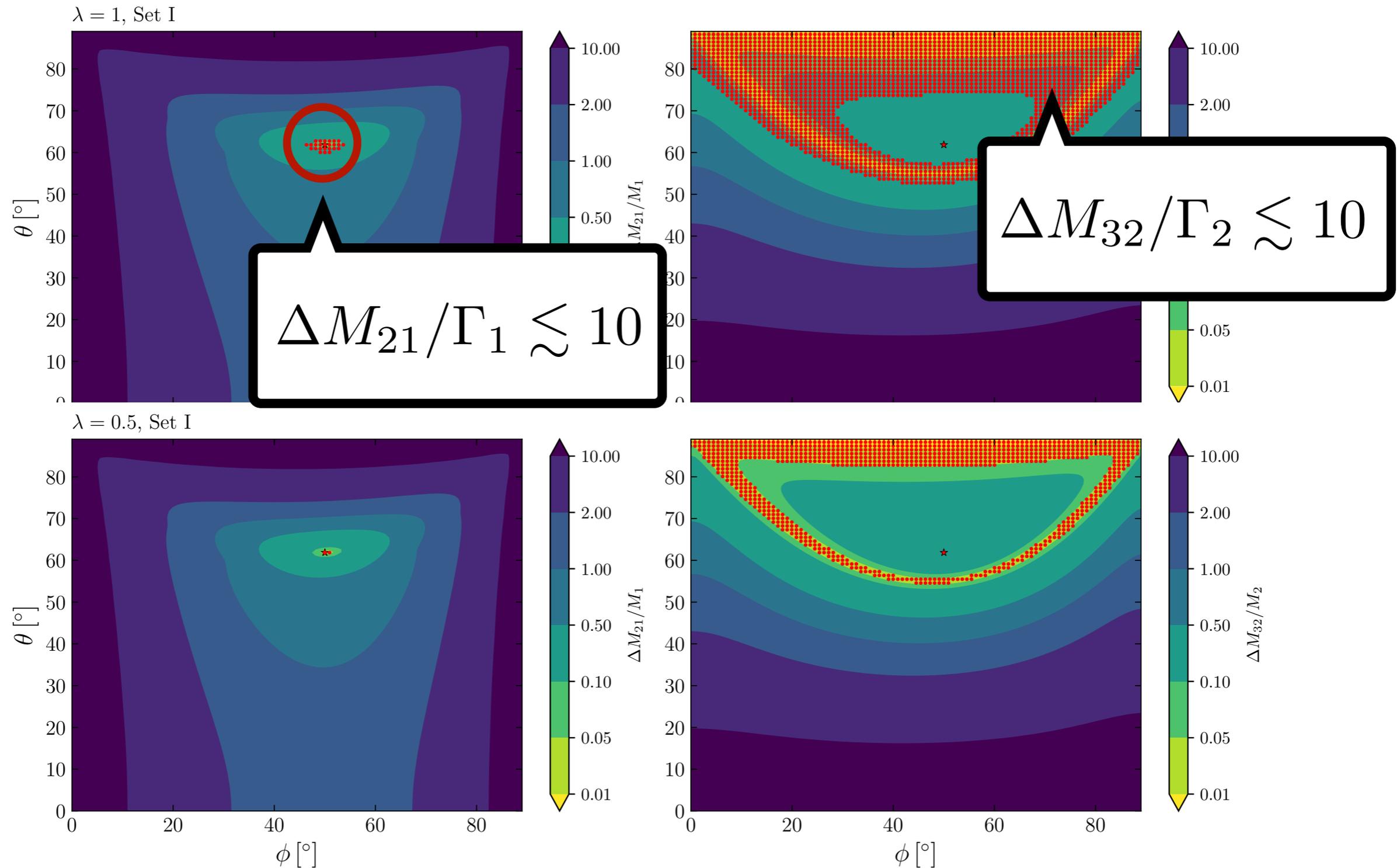
$$\text{Cf) } \lambda = 0.5$$



NuFIT Collaboration, NuFIT v5.2, <http://www.nu-fit.org>.

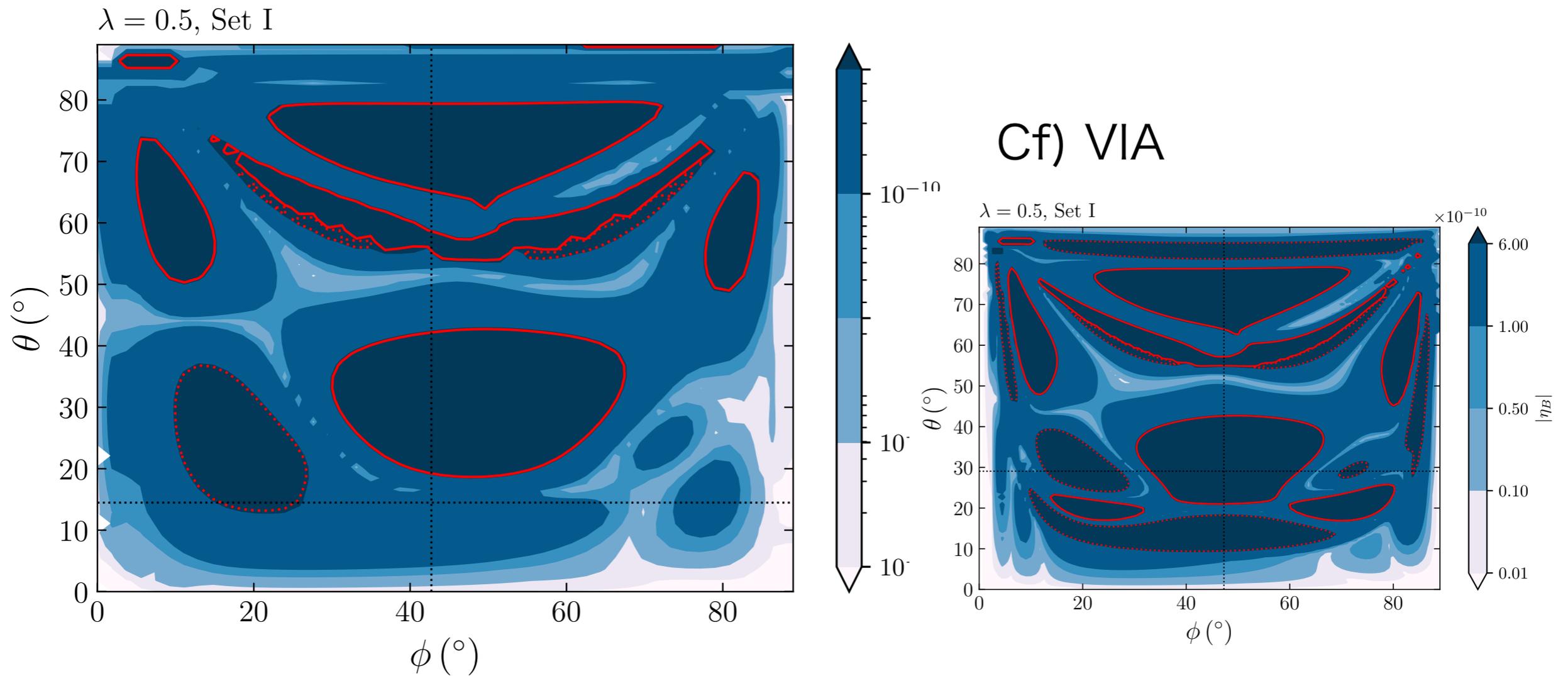
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# Impact of Resonance Effects



# Dependence of initial condition

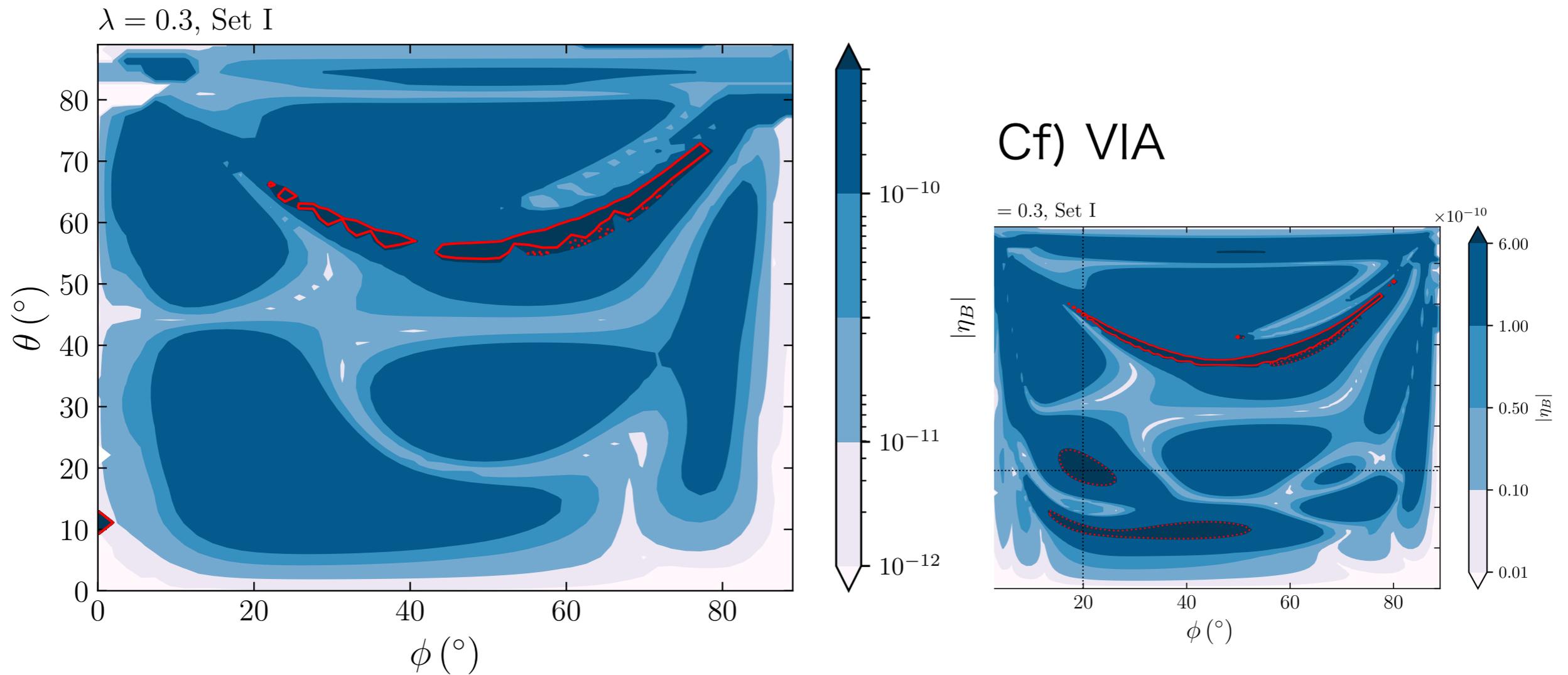
When we take thermal initial abundance (TIA),



$$(\lambda_e, \lambda_\mu, \lambda_\tau) = \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$

# Dependence of initial condition

When we take thermal initial abundance (TIA),



$$(\lambda_e, \lambda_\mu, \lambda_\tau) = \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$